Lessons 5-1 & 5-2 Learning Check Key

2016

AP Calculus AB
Lessons 5-1 & 5-2 Learning Check (with review)

Name Heinl
Date

1. Determine if the Mean Value Theorem applies to $f(x) = 2 - x^2$ on the interval $\left[0, \sqrt{2}\right]$. If so, which of the following points (x-value only) is guaranteed to exists by the MVT?

   continuous

   (A) No, the Mean Value Theorem does not apply.

   (B) Yes; $x = -\frac{2}{\sqrt{2}} + 2$

   (C) Yes; $x = \pm\frac{1}{\sqrt{2}}$

   (D) Yes; $x = \frac{1}{\sqrt{2}}$

$A_{RoC} = f'(x)$

$\boxed{A_{RoC}}$

$\frac{f(\sqrt{2}) - f(0)}{\sqrt{2} - 0} =$

$\frac{0 - 2}{\sqrt{2} - 0} = \frac{-2}{\sqrt{2}}$

$f'(x) = -2x$

$\frac{-2x}{1} = \frac{-2}{\sqrt{2}}$

$-2\sqrt{2}\,x = -2$

$x = \frac{\cancel{-2}}{\cancel{-2\sqrt{2}}} = \frac{1}{\sqrt{2}}$

2. Find all possible functions with the given derivative $y' = \frac{9x}{\sqrt[5]{x}}$. $= 9x \cdot x^{-1/5} = 9x^{4/5}$

   a) $y = 5x^{9/5} + C$ b) $y = \frac{5}{9}x^{9/5} + C$

   c) $y = 45x^{1/5} + C$ d) $y = 9x^{1/5} + C$

$y = \frac{9}{\frac{9}{5}} x^{\frac{9}{5}} + C$

$y = 9 \cdot \frac{5}{9} x^{\frac{9}{5}} + C$

$y = 5x^{9/5} + C$

3. Suppose a function f is differentiable for all x and $f(0) = 0$. If $g(x)$ is defined as $g(x) = f(x)\cos x$, which of the following statements must be true? $(0,0)$

   I. There exists a number c in the interval $\left(0, \frac{\pi}{2}\right)$ such that $g'(c) = 0$.
   II. There exists a number c in the interval $\left(\frac{\pi}{2}, \pi\right)$ such that $g'(c) = 0$.
   III. There exists a number c in the interval $\left(-\frac{\pi}{2}, 0\right)$ such that $g'(c) = 0$.

   A) I only
   B) II only
   C) I and II only
   D) I and III only
   E) I, II, and III

$g(0) = f(0) \cdot \cos 0 = 0$

$g\left(\frac{\pi}{2}\right) = f\left(\frac{\pi}{2}\right) \cdot \cos\left(\frac{\pi}{2}\right) = 0$

$g\left(-\frac{\pi}{2}\right) = 0$

$g(\pi) = f(\pi) \cdot \cos(\pi) = -f(\pi)$

I & III have an A_{RoC} of 0, so $g'(c) = 0$ at some point on the interval.

II has an A_{RoC} of $\frac{-f(\pi)}{\frac{\pi}{2}}$, which does not have to equal zero.

OVER →

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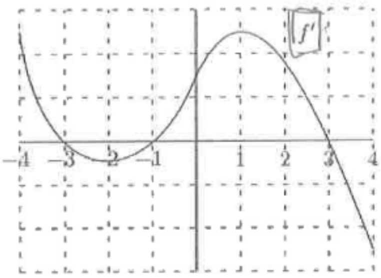
4.



The graph of the derivative of a function f is shown above. Which of the following are true about the original function f ?

I. f is increasing on the interval $(-2, 1)$. [X. f' is neg betw $(-2,-1)$]

II. f is continuous at $x = 0$. ✓ → [deriv. exists] Looking at it on the graph!

III. f has an inflection point at $x = -2$. ✓ Min or max of f'

A) I only

B) II only

C) ~~III only~~

D) II and III only

E) I, II, and III

5. The velocity of a particle moving along a horizontal axis is given by the function $v(t) = t^2 - \sin(2t)$ for $0 \le t \le 5$. If the position $s(t)$ of the particle at $t = 0$ is $\frac{1}{3}$, find an expression for the position of the particle at any time t.

$(0, \frac{1}{3}) \Rightarrow s(0) = \frac{1}{3}$

$s(t) = \frac{1}{3}t^3 + \cos(2t) \cdot \frac{1}{2} + C$

$\frac{1}{3} = \frac{1}{3}(0)^3 + \cos(0) \cdot \frac{1}{2} + C$

$\frac{1}{3} = \frac{1}{2} + C$

$\frac{1}{3} - \frac{1}{2} = C$

$-\frac{1}{6} = \frac{2}{6} - \frac{3}{6} = C$

$\boxed{x(t) = \frac{1}{3}t^3 + \frac{1}{2}\cos(2t) - \frac{1}{6}}$

6. Find the absolute extrema of $g(x) = \frac{x-1}{x+2}$ on $[-3, 2]$

$g'(x) = \frac{1(x+2) - 1(x-1)}{(x+2)^2} = \frac{x+2-x+1}{(x+2)^2} = \frac{3}{(x+2)^2}$

$g'(x) = 0$

$3 = 0$

(No zeros of $g'(x)$) but...

$x = -2$ is when $g'(x)$ is undefined

$f(-3) = \frac{-3-1}{-3+2} = \frac{-4}{-1} = 4$ max

$f(-2) = \frac{-2-1}{-2+2} \Rightarrow \varnothing$

$f(2) = \frac{2-1}{2+2} = \frac{1}{4}$ Min

$(-3, 4)$ Abs. Max

$(2, \frac{1}{4})$ Abs. Min

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6. Find the absolute extrema of $g(x) = \dfrac{x-1}{x+2}$ on $[0,2]$.

$$g'(x) = \frac{(x+2)(1)-(x-1)(1)}{(x+2)^2} = \frac{\cancel{x}+2-\cancel{x}-1}{(x+2)^2} = \frac{1}{(x+2)^2} \rightarrow (x+2)^2 = 0$$

$$x+2=0$$
$$\underline{-2\quad -2}$$
$$\boxed{\cancel{x=-2}}$$

not in interval

$$f(0) = \frac{-1}{2} \text{ min}$$

$$f(2) = \frac{2-1}{2+2} = \frac{1}{4} \text{ max}$$